



## ADJUSTMENT OF REAL GAS EQUATIONS TO CALCULATION OF GAS JET PUMPS

**Olena Babenko**

*«Research and Design Institute of Gas Transportation of JSC Ukrtransgaz»,  
Kharkiv, Ukraine, E-mail: babenko-eo@utg.ua  
PhD in technology, Lead Engineer*

When choosing the suitable type of gas equipment and further calculating its performance characteristics, equations based on the ideal gas law are used most widely. Equations for imperfect gas are less common in practice due to the fact that the calculations and the application in practice are more complex. This paper revisits some imperfect gas equations and gives recommendations on how to apply them based on an example of jet pump calculation. The paper illustrates how the calculation results for perfect and imperfect gas vary more as the pressure rises.

One of the main outcomes of the study is the ability to take account of the extra losses in the jet pump caused by the nature of real gas. This will make it more simple to commence gas equipment and will lead to more accurately predicted performance characteristics for certain conditions of operation, which, in turn, allows for a higher efficiency of the commenced gas jet pumps.

*Keywords:* imperfect gas, real gas, isentropic flow equations, jet pump

## АДАПТАЦІЯ СПІВВІДНОШЕНЬ РЕАЛЬНОГО ГАЗУ ДЛЯ РОЗРАХОВУВАННЯ ГАЗОВИХ ЕЖЕКТОРІВ

**О.О. Бабенко**

*Філія «Науково-дослідний інститут транспорту газу» АТ «Укртрансгаз»,  
Харків, Україна, E-mail: babenko-eo@utg.ua  
Кандидат технічних наук, провідний інженер*

Для підбирання газового обладнання й подальшого розраховування режиму його роботи часто використовують рівняння ідеального газу. Рівняння, що описують реальні гази, складні в обчислюванні та використовуванні на практиці. У статті уточнено деякі рівняння реального газу та надано рекомендації, як застосовувати їх до розраховування газового ежектора. Продемонстровано, що вплив реального газу на розрахунок ежектора стає значнішим за умови високого тиску.

У результаті, отриманий метод дасть змогу врахувати додаткові втрати в ежекторі, пов'язані з властивостями реального газу. Це спростить уведення в експлуатацію газового обладнання та підвищить точність розрахунку режиму роботи для певних умов застосовування. Своєю чергою це підвищить фактичну ефективність газоструйного обладнання.

*Ключові слова:* реальний газ, співвідношення реального газу (газодинамічні функції), газовий ежектор



# АДАПТАЦИЯ СООТНОШЕНИЙ РЕАЛЬНОГО ГАЗА ДЛЯ РАСЧЕТА ГАЗОВЫХ ЭЖЕКТОРОВ

**Е.О. Бабенко**

*Филиал «Научно-исследовательский институт транспорта газа» АТ «Укртрансгаз»,  
Харьков, Украина, E-mail: babenko-eo@utg.ua  
Кандидат технических наук, ведущий инженер*

Для подбора газового оборудования и дальнейшего расчета режима его работы часто используются уравнения для идеального газа. Уравнения, описывающие реальные газы, имеют определенные сложности в вычислении и использовании на практике. В данной работе уточнены некоторые уравнения реального газа и даны рекомендации применения их в расчетах на примере газового эжектора. Продемонстрировано влияние реального газа на расчет эжектора при переходе в область высокого давления.

В результате, полученный метод позволит учесть дополнительные потери в эжекторе, связанные со свойствами реального газа. Это позволит упростить ввод в эксплуатацию газового оборудования и повысит точность расчета режима работы для определенных условий работы. В свою очередь, это обеспечит более высокую фактическую эффективность газоструйного оборудования.

*Ключевые слова:* реальный газ, соотношения реального газа (газодинамические функции), газовый эжектор

## ***Introduction***

The dynamic behavior of gas that has certain thermodynamic properties is determined by its velocity, density and pressure. We can model this with a system of equations. Such a system incorporates the differential equations for the mass conservation, impulse conservation and energy conservation laws. Due to how complex the actual physical and chemical processes are, it is often not practically possible to solve such a system of equations with the use of analytical or numerical methods. Furthermore, depending on if the velocity of the gas flow is below, above or equal to the critical velocity, an entirely different mathematical apparatus is required. Yet another complication is the need to solve partial derivative equations. One of the ways around these complexities is to base the analytical solution on specially crafted approximations. This allows us to dramatically simplify the required mathematical treatment while identifying the key characteristics of the flow. The existing methods, however, are based on the assumptions that impair the precision of the calculation. This



leads to reduced efficiency of the jet pump and prevents the organization from selecting the proper device for certain operating conditions, which is discussed in an article by the author (Chystiak, 2005). The gas industry extensively employs jet pump devices, for example, to inject gas into underground storages, dispose of gas, or jointly use wells that vary in operating pressure.

### ***Recent literature review***

Analytical methods are still the most important approach to jet pump calculation and matching. The model of the gas flow processes should involve both the equations of continuity and the energy equation working as a whole. Solving this system of equations by analytical methods would be almost impossible provided the assumption of non-isothermal processes. Application of such a system would most definitely take a good deal of complex software and powerful hardware. Subsequently, making the assumption about an isometric nature of the gas dynamic processes has become the commonly adopted way of applying the analytical methods.

As far as calculation of gas jet pumps is concerned, the lead is kept by the methods based on the classical equations of mass conservation, energy conservation and impulse conservation for ideal gas (Schacht, Slobodkina, 2006). (Hicks, 1960) notes, however, that the methods based on a gas dynamics analysis of the jet pump processes are not representative of how various input parameters influence the performance of the jet pump. For the devices operating with supersonic flows, the recommendation is made to give special consideration to the effects of compressibility and shock waves.

According to (Rajzman, 1995), the processes in a gas jet pump should be modelled based on a two-dimensional gas flow model. In certain cases, the mathematical ramifications take such calculations to beyond the limits of what is practically possible. The generally accepted approach is to treat ejector operation



according to the theory of unidimensional gas flow. This paper is based on the (Sokolov, 1989) method. The method is based on the mass conservation law, energy conservation law and the momentum equation (Rajzman, 1995). However, similar to the classic calculation methods based on the isentropic flow equations (Sokolov, 1989), this approach to calculate jet pumps is founded on the ideal flow assumption.

The thermodynamic properties of the real gases can be defined when the equation of state is known. Based on that, the density, the compressibility factor, the ratio of specific heats and the dynamic viscosity factor are calculated. This is discussed in greater detail in another article by the author (Chystiak, 2010). Analysis of the isentropic flow equations for both real and ideal gas is conducted in (Shehtman, 1988). The ideal gas equations are founded on the Clapeyron's equation, as well as on the fact that both the isobaric and isochoric heat capacity are considered to not be influenced by the pressure. This means the ratio of specific heats used to model ideal gas is not influenced by the pressure either. Both enthalpy and intrinsic energy are influenced exclusively by temperature. That said, the compressibility factor, enthalpy and intrinsic energy of various gases in the single-phase conditions are subjected to considerable changes. Because of that, we cannot apply the gas dynamic equations nor the reference data for ideal gas isentropic flow equations in order to precisely calculate real gas flows, unless the relevant adjustments are made.

(Chystiak, 2012) suggests a comparison study of the real and ideal gas equations. The paper discusses the isentropic flow equations, compressibility factor and other parameters.

### ***Data and method of the study***

The use of isentropic flow equations for real gas instead of those for ideal gas will allow for higher operational efficiency of the gas equipment implemented based on the relevant calculations.



In the field of identification of isentropic flow equations for ideal gas, the classics of gas dynamics literature mostly feature only certain reference data sheets and/or diagrams as opposed to formulas. Reference data sheets that represent isentropic flow equations for ideal gas are good for practical use, whereas the formulas for ideal gas are much more simple than those for real gas (Chystiak, 2012).

Reference data gives an opportunity to instantly get a ballpark of the value of the function, i.e. to prognosticate the behavior of the flow. This eliminates the need for labor-intensive calculations and the need for any other handbooks providing the input data for this particular kind of gas, such as heat capacity or compressibility factor. This is necessary to acquire the input data for calculation. In addition, the reference data (Shehtman, 1988) takes account of the little changes in pressure and temperature, using specially designed algorithms. According to (Shehtman, 1988), the suggestion is to use the following isentropic flow equations for real gas:

$$\tau = \frac{T}{T_0} = \frac{\alpha_0}{\alpha} \left( 1 - \frac{\lambda^2}{1 + \beta_*} \right) \quad (1)$$

$$\pi = \frac{p}{p_0} = \left[ \frac{\alpha_0}{\alpha} \left( 1 - \frac{\lambda^2}{1 + \beta_*} \right) \right]^{\frac{x}{x-1}} \quad (2)$$

$$\varepsilon = \frac{\rho}{\rho_0} = \frac{z_0}{z} \left[ \frac{\alpha_0}{\alpha} \left( 1 - \frac{\lambda^2}{1 + \beta_*} \right) \right]^{\frac{1}{x-1}} \quad (3)$$

where  $\tau$  is the temperature ratio;  $\pi$  is the pressure ratio;  $\varepsilon$  is the density ratio;  $p$  is the gas pressure, MPa;  $T$  is the gas temperature, K;  $\rho$  is the gas density, kg/m<sup>3</sup>;  $\alpha$  and  $\beta$  are dimensionless thermodynamic functions;  $\lambda$  is the velocity ratio (the ratio of the flow velocity and the critical velocity);  $x$  is the ratio of specific heats;  $z$  is the compressibility factor. All quantities indexed with "0" are for the decelerating flow. All quantities indexed with "\*" are for the flow at critical conditions.

According to (Shehtman, 1988), the isentropic flow equation of the normalized flow rate for real gas is a ratio of the mass velocity of the flow in this particular section and the mass velocity of the flow at the throat:



$$q = \frac{w\rho}{a_*\rho_*} = \frac{z_*}{z} \left[ \frac{\alpha_*}{\alpha} \frac{1 + \beta_* - \lambda^2}{\beta_*} \right]^{\frac{1}{\gamma-1}} \lambda \quad (4)$$

where  $w$  is the velocity of the gas flow, m/s;  $a$  is the velocity of the flow, m/s.

The isentropic flow equations involve the inputs such as compressibility factor or dimensionless thermodynamic functions that characterize the behavior of the flow at certain operating conditions and velocity. The slight change in these factors caused by the change in the velocity of the flow was dealt with in the reference data (Shehtman, 1988) by means of the specially designed algorithms and programs.

In gas dynamics, a concept of a ‘representative’ point can be used. The first representative point is when the velocity ratio equals zero ( $\lambda = 0$ ); at this point the flow decelerates. When the velocity ratio equals one ( $\lambda = 1$ ), the flow velocity equals the critical velocity; this is the second representative point we want to look at. Isentropic flow equations in these two points should have specific, predetermined values. However, an observation can be made that these formulas do not always give the precise match on the expected value in a ‘representative’ point. This contradicts the nature of the model describing the physical process.

When the flow reaches the critical velocity ( $\lambda$  equals 1), the mass velocity of the flow equals the mass velocity of the flow at the throat;  $q$  equals 1. However, when the math is done by hand, the slight changes in the factors of  $z$  and  $\alpha$  are neglected. At a critical velocity of the flow ( $\lambda=1$ ), the normalized mass velocity ( $q$ ) will be as follows:

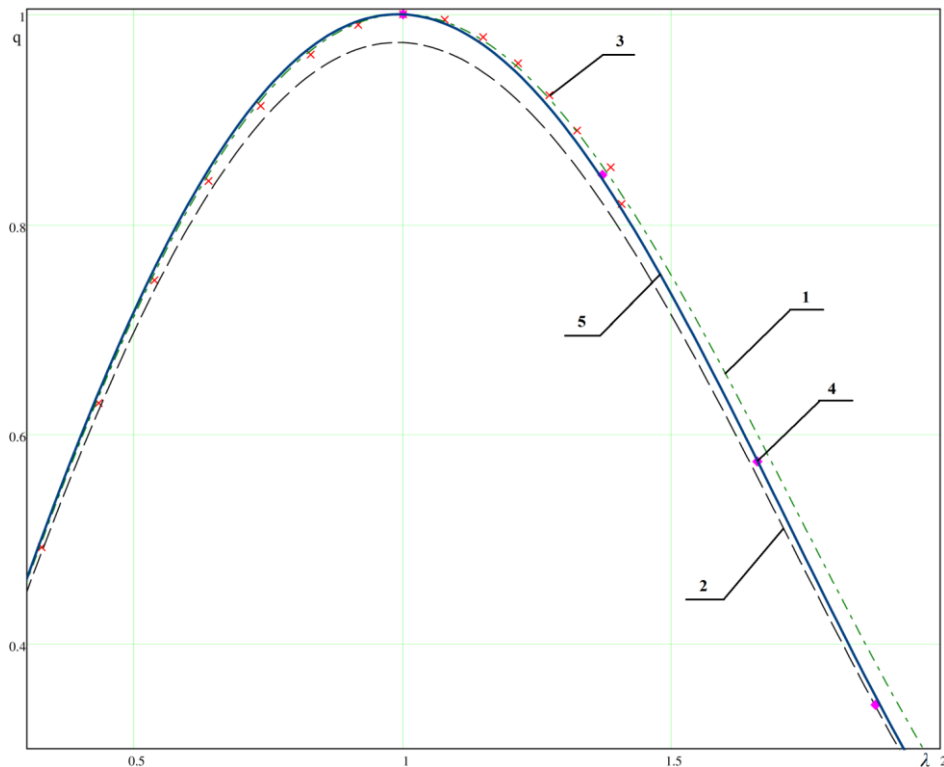
$$q_* = \frac{z_*}{z_*} \left[ \frac{\alpha_*}{\alpha_*} \frac{1 + \beta_* - 1^2}{\beta_*} \right]^{\frac{1}{\gamma-1}} 1 = 1 \quad (5)$$

The values of  $z$  and  $\alpha$  are introduced to the formula. These factors characterize the state of the flow at certain operating conditions, when the velocity of the flow is unknown. Thanks to the contribution of the factors of  $z$  and  $\alpha$ , when doing the math by hand, the function  $q$  for  $\lambda=1$  will not be equal to the value of 1.

## Key findings

Based on the formula for normalized flow rate, the normalized flow rate was expressed as a function of the velocity ratio applicable for methane, at the pressure of 8 MPa and temperature of 330K.

While plotting the graph (Fig. 1), both reference data and formulae for different methods were used.



**Figure 1.** The comparison of  $q$  (normalized mass velocity) under different  $\lambda$  (velocity ratio). Inputs:  $p = 8$  MPa,  $T = 330$  K.

Key: 1 – calculated by the ideal gas method; 2 – calculated by the Shehtman method (Shehtman, 1988); 3,  $\times$  – reference data (Malkhanov, 1981); 4,  $\blacklozenge$  – reference data (Shehtman, 1988); 5 – calculated by the modified formulas suggested in this paper (6-9).

The dimensionless factors  $z$  and  $\alpha$  – formula (4) – must change in a dynamic way as soon as the flow reaches the critical characteristics. For instance, when  $\lambda$  equals 1,  $z$  and  $\alpha$  must equal  $z_*$  and  $\alpha_*$  respectively, which is not reflected in this formula. Nonetheless, this formula aligns well with the reference data, given the velocity ratio of 2 or more for this specific isentropic flow equation. If  $z$  and  $\alpha$  are replaced by their



critical counterparts  $z_*$  and  $\alpha_*$ ,  $q$  will equal 1 when  $\lambda$  equals 1. If this thinking is extended upon other isentropic flow equations, the following can be derived:

$$\tau = 1 - \frac{\lambda^2}{1 + \beta_*} \quad (6)$$

$$\pi = \left( 1 - \frac{\lambda^2}{1 + \beta_*} \right)^{\frac{x}{x-1}} \quad (7)$$

$$\varepsilon = \left( 1 - \frac{\lambda^2}{1 + \beta_*} \right)^{\frac{1}{x-1}} \quad (8)$$

$$q = \left( \frac{1 + \beta_* - \lambda^2}{\beta_*} \right)^{\frac{1}{x-1}} \quad (9)$$

The further  $\lambda$  increases from the value of 1, the more visible the deviation on the chart is between the graph for formula (9) and the graph for the formula based on ideal gas. An attempt to find in-between values by linear interpolation of the reference data would lead to a significant loss of precision. As  $\lambda$  (the velocity ratio) increases, the values calculated by formula (4) start aligning with the reference data.

Other isentropic flow equations also require taking account of the velocity of the flow in order to determine the factors for the calculation formulas. When 1) the velocity ratio is relatively low and 2) the velocities of the gas flows are close to the critical velocity, the recommendation would be to use the simplified formulas (6-9). When  $\lambda$  (the velocity ratio) is more than 2.5, the use of the standard formulas for real gas (1-4) is recommended.

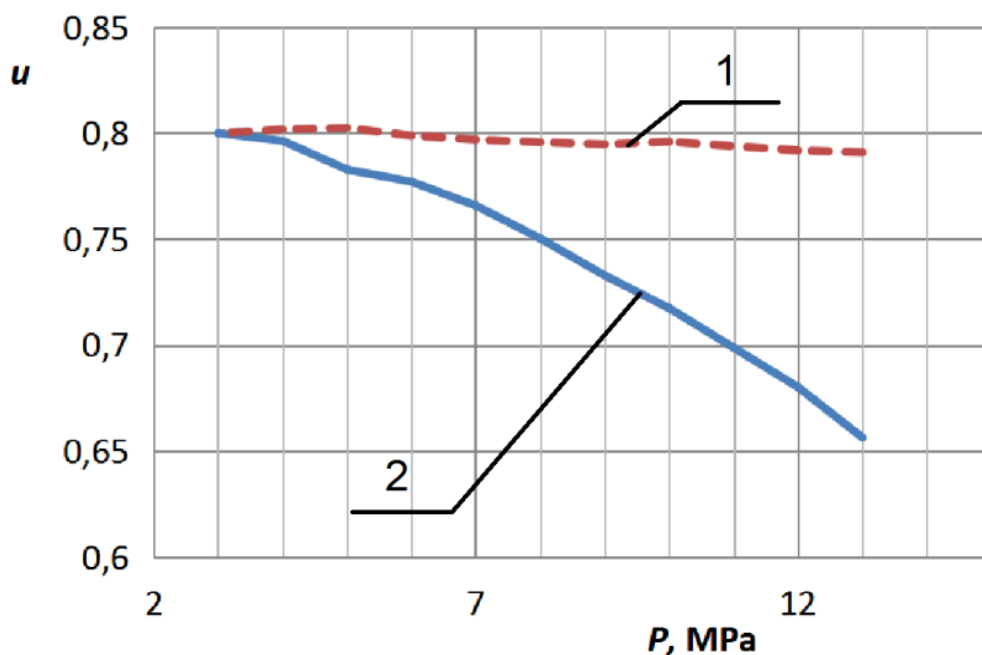
The use of the simplified isentropic flow equations is justified when calculation of jet pumps is made for a flow with a velocity that is approaching the critical value. The use of the real gas isentropic flow equations instead of the ones for ideal gas allows for elaboration of engineering methods that result in a more accurate calculation of fluid dynamics devices, such as gas pumps.

The foundation for this research is laid by a method for gas jet pump calculation that is based on the ideal gas equations (Sokolov, Zinger, 1989). As soon as the



calculations embrace the equations for real gas (1-4) and (5-9), the additional losses in the jet pump will be dealt with when operating at higher pressures.

Figure 2 shows the comparison of the ejection factor calculation methods. Ejection factor defines how efficiently the gas device is operating. It is determined as a ratio between the flow rates of the low-pressure (secondary) fluid and the high-pressure (motive or primary) fluid.



**Figure 2.** Comparison of the calculation results for ejection factor  $u$  depending on the motive flow pressure  $p$  at the intake of the jet pump.  
Key: 1 – Sokolov method (Sokolov, Zinger, 1989); 2 – the suggested method, which is based on the real gas equations.

As a result, a change in the ejection factor can be observed depending on the change of operational conditions. The difference between the ejection factors calculated based on the ideal gas method and the suggested method tends to increase as the pressure increases.

Fig. 2 provides evidence that with a change in pressure from 6 MPa to 10 MPa, the difference in the calculated gas jet pump ejection factor for ideal and real gas increases from 5% to as much as 30%. This translates into a difference in the geometry



of the unit of up to 10%. When the pressure is under 1.2 MPa, the difference in the calculated ejection factors with use of the suggested and the classic method can reach the 10% mark. The higher the pressure the more noticeable the influence of the compressibility factor is, as far as the operational characteristics of a jet pump go.

### ***Conclusions***

Calculations based on imperfect gas equations should be treated as those of approximate nature since they do not always meet the modern requirements to the precision of gas equipment. The field of application for such calculation methods is limited due to a number of assumptions in the primary gas dynamic equations leading to inaccuracies. The lack of knowledge on temperature gas dynamics and heat and mass transfer processes for some types of stream flows results in reduced actual efficiency of the units compared to the expected figures calculated during the design phase.

The methods have inaccuracies, which means they have to be double-checked by practical observations. This hinders the application of jet pumps, especially when the pressure of the motive fluid is high. High motive fluid pressure is often the case when jet pumps are used to disposition gas, increase pipeline flow capacity, enable the simultaneous operation of natural gas fields with dramatically different reservoir pressures, etc.

Furthermore, jet pumps have a number of issues when it comes to practical application. During installation, a jet pump requires additional configuration in order to meet the predicted performance characteristics. Initial commissioning testing of jet pumps involves a certain amount of trial and error as the theoretically calculated performance characteristics are unlikely to be met at the first attempt with sufficient precision. If the actual performance characteristics significantly vary from the theoretical optimal values, the equipment may operate inconsistently or even fail to operate at all, ceasing the gas transition process completely.

When multiple wells of different operating pressure are used in combination, the objective is to calculate the parameters of the jet pumps so as to allow for the most efficient pipeline operation. When compared with the other types of pumps, one of the main downsides of a jet pump is lower operational efficiency. This is why it is



extremely important to make a precise calculation and achieve the maximum possible efficiency of a jet pump. On the contrary, use of the classic methods to calculate the parameters of a jet pump may lead to a lower actual performance upon installation. This means the actual cost of gas transportation will be higher. If the installed jet pump does not operate with the projected efficiency, a change in the gas flow parameters on the intake will likely be necessary in order to obtain the expected performance characteristics. This can be achieved by increasing either the pressure or the flow rate, both of which resulting in extra expenses. This again spotlights the need to improve the gas jet pump calculation methods based on the properties of the real gas.

The isentropic flow equations for real gas have been analyzed, the recommendations have been made on how to apply the equations to calculate the processes of gas dynamics for the representative values of the velocity ratio.

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